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On the effects of a gravitational field on the turbulent transport of heat and momentum

By B. E. LAUNDER
Mechanical Engineering Department, Imperial College, London

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This paper suggests a simple way of including gravitational effects in the pressure-containing correlations that appear in the equations for the transport of Reynolds stress and heat flux. The predicted changes in structure due to the gravitational field are shown to agree closely with Webster's (1964) measurements in a stably stratified shear flow.

1. Introduction

There is currently much activity in developing turbulence closures based on approximated sets of transport equations for the Reynolds stresses $\overline{u_i u_j}$ and, where appropriate, for the heat-flux correlations $\overline{u_i T'}$. This is the simplest level of closure at which one may hope to account for the effects of external force fields on the turbulence without the need to introduce *ad hoc* modifications. Moreover, from the computational point of view, it happens to be the most comprehensive kind of model that can be handled at present in making numerical computations of practically important flows. Thus second-order closures seem to offer the one kind of turbulence model with the prospect of becoming both adequately general and tolerably simple.

Of course, the exact transport equations for $\overline{u_i u_j}$ and $\overline{u_i T'}$ contain several correlations of fluctuating quantities that cannot be represented exactly. Of these the pressure–strain correlation in the $\overline{u_i u_j}$ equation and the pressure–temperature correlation in the heat-flux equation are of decisive importance. For simple shear flows in the absence of force fields, three independent researches (those reported by Naot, Shavit & Wolfshtein 1973; Lumley & Khajeh Nouri 1973; Launder, Reece & Rodi 1975) have led to virtually identical proposals for the pressure–strain correlations, so an encouraging unanimity is emerging. However, when force fields are present (and, in the case of the pressure–temperature correlation, even for simple flows) there has been so little comparison of computation with experiment that all proposals must be regarded as tentative.

In considering the problem of turbulence affected by buoyancy, most workers have assumed there is no direct influence of gravity on the important pressure-containing correlations in the stress and heat-flux equations (e.g. Lumley 1972; Donaldson, Sullivan & Rosenbaum 1972; Daly 1972). The present note argues that such an influence should in fact be included; a generalization of a simple model used by Launder *et al.* (1975) in computing a number of non-buoyant thin shear flows is shown to give approximately the correct effects.

2. Analysis

The equations governing the transport of Reynolds stress $\overline{u_i u_j}$ and heat flux $\overline{u_i T'}$ in a turbulent fluid may be written as

$$\frac{D\overline{u_i u_j}}{Dt} = -\underbrace{\left\{\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}\right\}}_{\text{generation by shear}} - \underbrace{\frac{\alpha}{T}\{\overline{u_i T'} g_j + \overline{u_j T'} g_i\}}_{\text{generation by buoyant forces}}$$

$$- \underbrace{2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}_{\text{viscous dissipation}} + \underbrace{\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}}_{\text{pressure scrambling}}$$

$$- \underbrace{\frac{\partial}{\partial x_k}\left[\overline{u_i u_j u_k} + \delta_{ik}\frac{\overline{u_j p}}{\rho} + \delta_{jk}\frac{\overline{u_i p}}{\rho}\right]}_{\text{diffusive transport}}, \tag{2.1}$$

$$\frac{D\overline{u_i T'}}{Dt} = -\underbrace{\left\{\overline{u_i u_k}\frac{\partial T}{\partial x_k} + \overline{u_k T'}\frac{\partial U_i}{\partial x_k}\right\}}_{\text{mean-field generation}} - \underbrace{\frac{\alpha \overline{T'^2} g_i}{T}}_{\text{buoyant generation}}$$

$$- \underbrace{(\lambda + \nu)\overline{\frac{\partial T'}{\partial x_k}\frac{\partial u_i}{\partial x_k}}}_{\text{dissipation}} + \underbrace{\overline{\frac{p}{\rho}\frac{\partial T}{\partial x_i}}}_{\text{pressure scrambling}} - \underbrace{\frac{\partial}{\partial x_k}\left(\overline{u_i T' u_k} + \frac{\overline{pT'}}{\rho}\right)}_{\text{diffusive transport}}, \tag{2.2}$$

where upper and lower case u's denote fluctuating and mean velocities, T and T' fluctuating and mean temperatures, ρ the mean fluid density, p the instantaneous pressure fluctuation about its mean value, g_i the component of gravitational acceleration in the direction x_i and α the dimensionless volumetric expansion coefficient of the fluid. Apart from the neglect of molecular diffusion, the above transport equations are a direct consequence of the equations of motion and the first law of thermodynamics. However, apart from generation terms, al l the processes affecting the level of $\overline{u_i u_j}$ and $\overline{u_i T'}$ introduce unknown correlations of fluctuating quantities. These, of course, need to be approximated in terms of the main dependent variables in order to close the equations. Here it is the pressure scrambling terms in the two equations to which attention is mainly given.

First, notice that taking the divergence of the momentum equation for u_i produces the following Poisson equation for the pressure fluctuation:

$$\frac{1}{\rho}\frac{\partial^2 p}{\partial x_i^2} = -\left(\frac{\partial^2(u_i u_j - \overline{u_i u_j})}{\partial x_j \partial x_i} + 2\frac{\partial U_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + \alpha\frac{\partial T'}{\partial x_i}\frac{g_i}{T}\right). \tag{2.3}$$

Equation (2.3) shows that pressure fluctuations arise from three agencies: from purely turbulence interactions, effects due to mean strain and the effects of temperature fluctuations in a gravitational field. It seems reasonable, therefore, that approximations for the correlations containing p should mirror these three contributions to the level of pressure fluctuations.

Let us for the moment consider the approximation of the pressure–strain correlation in (2.1) for the case of non-buoyant flows. Recently, Launder *et al.*

(1975), having made calculations of a number of free shear flows using a closed-form approximation of (2.1), concluded that the following simple form produced approximately the right effect of the pressure–strain interaction:

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} = -c_1\frac{\epsilon}{k}(\overline{u_iu_j}-\tfrac{2}{3}\delta_{ij}k)-c_2(P_{ij}-\tfrac{2}{3}\delta_{ij}P). \tag{2.4}$$

The scalar quantities k, P and ϵ are respectively the turbulence kinetic energy $\frac{1}{2}\overline{u_iu_i}$, its rate of generation by mean strain and its rate of dissipation by viscous action. P_{ij} is an abbreviation for the mean-strain generation rate of $\overline{u_iu_j}$ in (2.1) and the coefficients c_1 and c_2 are taken as constants. The two terms on the right side of (2.4), the first composed of correlations of just fluctuating quantities and the second containing mean-strain terms, may be taken as approximating the effects of the first two terms on the right side of (2.3).

The 'turbulence interaction' part of (2.4) was due originally to Rotta (1951), who argued that 'collisions' among the energy-containing eddies would promote a return to isotropy at a rate proportional to the prevailing level of anisotropy. The second term in (2.4) was first proposed by Naot, Shavit & Wolfshtein (1970) (though, rather curiously, as a *replacement* for Rotta's term, not as an additional contributor). The idea is that pressure fluctuations induced by mean strain will act to make the production tensor more isotropic – a plausible enough idea. Direct support for the form adopted is provided by the more comprehensive analyses of Launder *et al.* (1975) and, particularly, of Naot *et al.* (1973). The mean-strain part of (2.4) emerges from these analyses as the leading, and substantially the most important, term of more complicated expressions for

$$\overline{(p/\rho)\,(\partial u_i/\partial x_j+\partial u_j/\partial x_i)}.$$

Although Rotta (1951), Townsend (1954) and numerous workers since have shown the importance of mean-strain influences on the pressure–strain correlation, many of the proposals for closing (2.1) have included only the first term in (2.4) (the omission can be disguised over a narrow range of flows by suitably increasing the magnitude of c_1). Not surprisingly, therefore, the same forms were carried over for use in buoyant shear flows (e.g. Lumley 1972; Donaldson *et al.* 1972). However, once one accepts that mean-strain generation should appear in the approximation of $\overline{p\,\partial u_i/\partial x_j}$, it is difficult not to conclude that generation terms arising from buoyancy should be included. A rational extension of (2.4) to the case of buoyant flow, the main proposal of the present note, is that P_{ij} and P should now be taken to stand for the *total* generation of $\overline{u_iu_j}$ and k due to the combined effects of shear and buoyancy, i.e.

$$\left.\begin{aligned} P_{ij} &\equiv -\left\{\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}+\overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right\}-\frac{\alpha}{T}\{\overline{T'u_i}g_j+\overline{T'u_j}g_i\} \\ \text{and}\qquad P &\equiv -\left\{\overline{u_iu_k}\frac{\partial U_i}{\partial x_k}+\frac{\alpha}{T}\overline{u_iT'}g_i\right\}. \end{aligned}\right\} \tag{2.5}$$

Consistently, the same practice is adopted in approximating the pressure scrambling term in the $\overline{u_i T'}$ equation. Nearly every worker who has made closure approximations for (2.2) has assumed that

$$\overline{(p/\rho)(\partial T'/\partial x_i)} = -c_{1T}(\epsilon/k)\overline{u_i T'}. \tag{2.6}$$

This is the counterpart of the first term on the right side of (2.4) and again neglects gravitational and mean-strain effects on p. In parallel with (2.4), therefore, (2.6) is modified to

$$\overline{(p/\rho)\,\partial T'/\partial x_i} = -c_{1T}(\epsilon/k)\overline{u_i T'} - c_{2T}P_{iT}. \tag{2.7}$$

At this point let us note that formally

$$\frac{\overline{Du_i T'}}{Dt} = \frac{\overline{u_i DT'}}{Dt} + \frac{\overline{T' Du_i}}{Dt} \tag{2.8}$$

and that pressure fluctuations appear only in the transport equation for u_i, not T'. Accordingly, P_{iT} is interpreted as the generation of $\overline{u_i T'}$ arising from the term $\overline{T'Du_i/Dt}$, i.e.

$$P_{iT} = -\left\{\overline{u_k T'}\frac{\partial U_i}{\partial x_k} + \frac{\alpha\overline{T'^2}g_i}{T}\right\}.$$

In §3 the implications of (2.4) and (2.7) are compared with experimental data in a near-equilibrium shear flow where transport effects on the stresses and heat fluxes should be negligible. If, therefore, transport terms are neglected in (2.1) and the dissipative motions are assumed isotropic, i.e.

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \tfrac{2}{3}\delta_{ij}\epsilon, \tag{2.9}$$

the following simple algebraic expression emerges for the Reynolds stress:

$$(\overline{u_i u_j} - \tfrac{2}{3}\delta_{ij}k)/k = \phi(P_{ij} - \tfrac{2}{3}P\delta_{ij})/\epsilon, \tag{2.10}$$

where $\phi \equiv (1-c_2)/c_1$.

The value of ϕ may be obtained from the very carefully generated (isothermal) homogeneous shear flow of Champagne, Harris & Corrsin (1970). The experiments suggest that $\overline{u_1^2}$, the streamwise normal stress, is approximately $0{\cdot}93k$. Thus from (2.10)

$$(\overline{u_1^2} - \tfrac{2}{3}k)/k = \tfrac{4}{3}\phi \tag{2.11}$$

and therefore the value of ϕ is taken as $0{\cdot}20$. There is of course no necessity for present purposes to prescribe c_1 and c_2 separately; however it is of interest to note that, with c_2 given the value $0{\cdot}6$ (implying $c_1 = 2{\cdot}0$), equation (2.4) exactly satisfies Crow's (1968) result for the sudden distortion of isotropic turbulence, viz.

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} = -\tfrac{2}{5}k\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right). \tag{2.12}$$

We now obtain the counterpart of (2.1) for the heat-flux equation. In parallel with the treatment of (2.1) we neglect transport terms in (2.2) and assume isotropy of the fine-scale motion. Thus

$$(\lambda+\nu)\overline{\frac{\partial T'}{\partial x_k}\frac{\partial u_i}{\partial x_k}} = 0. \tag{2.13}$$

Now, the buoyancy term in (2.2) contains $\overline{T'^2}$, which remains as an unknown at present. Following Monin (1965) this quantity is obtained by equating the generation and dissipation rates of $\overline{T'^2}$:

$$\lambda \overline{\frac{\partial T'}{\partial x_k} \frac{\partial T'}{\partial x_k}} = -\overline{u_k T'} \frac{\partial T}{\partial x_k}. \tag{2.14}$$

The dissipation rate is then assumed to be proportional to $\epsilon \overline{T'^2}/k$ and hence the following equation for the temperature fluctuations emerges:

$$\overline{T'^2} = -c'_T (k/\epsilon)\, \overline{u_k T'}\, \partial T/\partial x_k. \tag{2.15}$$

The coefficient c'_T is chosen by reference to the decay of temperature fluctuations behind a grid. Gibson & Schwarz (1963) found that the level of $\overline{T'^2}$ varied inversely as the three-halves power of distance behind the grid. Moreover a consensus of turbulence energy data in grid turbulence suggests that k decays as the $-1{\cdot}2$ power of distance. It may be shown that these two decay laws imply that c'_T should equal approximately 1·6 (i.e. $(1{\cdot}2/1{\cdot}5) \times 2$).

With all the above simplifications introduced, the following formula emerges for the heat flux in equilibrium flows:

$$-\overline{u_i T'} = \phi_T \frac{k}{\epsilon} \overline{u_k u_i} \frac{\partial T}{\partial x_k} + \phi'_T \frac{k}{\epsilon} \overline{u_k T'} \left\{ \frac{\partial U_i}{\partial x_k} - \frac{1{\cdot}6 \alpha g_i k}{\epsilon T} \frac{\partial T}{\partial x_k} \right\}, \tag{2.16}$$

where $\phi_T = 1/c_{1T}$ and $\phi'_T = \phi_T(1 - c_{2T})$. The empirical coefficients c_{1T} and c_{2T} are estimated by reference to Webster's (1964) nearly homogeneous shear flow (nominally the same flow as that examined by Champagne *et al.* (1970) but with the temperature increasing linearly with height) under essentially non-buoyant conditions. Under the stated conditions (2.16) gives the following equation for the heat flux in the direction of the temperature and velocity gradients:

$$-\overline{u_3 T'} = \frac{1}{c_{1T}} \frac{\overline{u_3^2} k}{\epsilon} \frac{\partial T}{\partial x_3}, \tag{2.17}$$

so the group $-k\overline{u_3^2}/c_{1T}\epsilon$ has the significance of an effective thermal diffusivity. Use of this equation to eliminate the mean temperature gradient from (2.15) yields

$$\overline{T'u}_3^2/\overline{T'^2}\,\overline{u_3^2} = 0{\cdot}63/c_{1T}. \tag{2.18}$$

Webster's data imply that the correlation on the left side of (2.18) is approximately 0·2; accordingly the value adopted for c_{1T} is 3·2.

The equation for the streamwise heat flux $-\overline{u_1 T'}$ takes the form

$$-\overline{u_1 T'} = \frac{k}{\epsilon c_{1T}} \overline{u_1 u_3} \frac{\partial T}{\partial x_3} + \frac{1 - c_{2T}}{c_{1T}} \frac{k}{\epsilon} \overline{u_3 T'} \frac{\partial U_1}{\partial x_3}. \tag{2.19}$$

Division of (2.19) by (2.17) leads after some manipulation to

$$\frac{\overline{u_1 T'}}{\overline{u_3 T'}} = \frac{\overline{u_1 u_3}}{\overline{u_3^2}} + \frac{1 - c_{2T}}{c_{1T}} \frac{k}{\overline{u_1 u_3}}. \tag{2.20}$$

Webster's measurements suggest that within narrow limits this correlation is close to unity when gravitational effects are negligible. With the value deduced

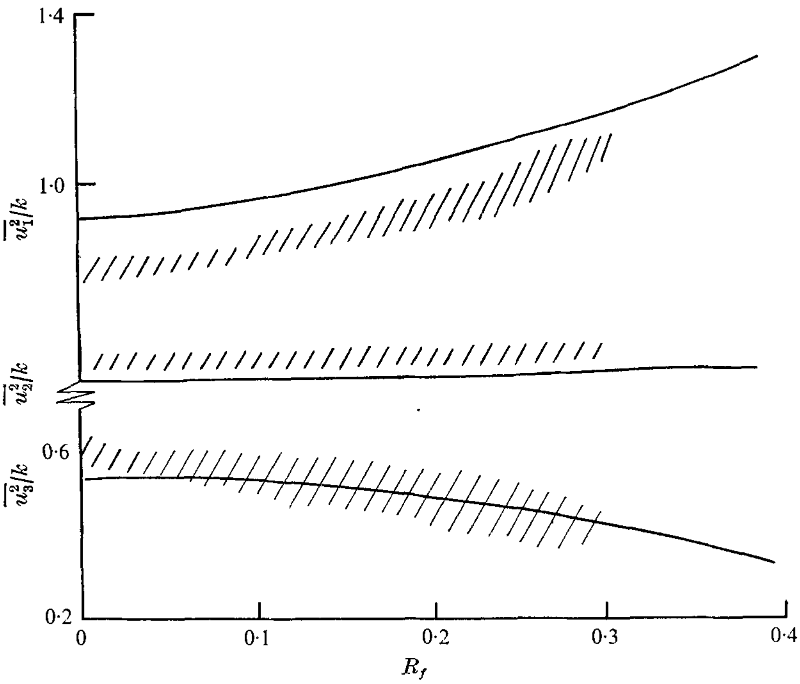

FIGURE 1. Dependence of normal-stress ratios on flux Richardson number. ////, band of Webster's data.

above for c_{1T} and those given by (2.10) for $\overline{u_1u_3}/k$ and $\overline{u_3^2}/k$, it is found that c_{2T} is approximately 0·5 (the precise value adopted); that is, according to the present proposals, 50 % of the heat flux generated by mean strain and buoyant actions is directly removed by pressure fluctuations.

3. Some implications for horizontal buoyant flows

Attention is now given to the effects of buoyancy on the turbulent flux. The co-ordinate x_3 is vertically upwards with the velocity U_1 and temperature varying only with x_3. Under these conditions (2.10) implies the following formulae for the normal-stress components:

$$\left.\begin{aligned} \overline{u_1^2}/k &= 0{\cdot}94 + 0{\cdot}41R_f/(1-R_f),\\ \overline{u_2^2}/k &= 0{\cdot}53,\\ \overline{u_3^2}/k &= 0{\cdot}53 - 0{\cdot}41R_f/(1-R_f),\end{aligned}\right\} \qquad (3.1)$$

where R_f is the flux Richardson number, i.e. the rate at which turbulence energy is removed by working against the gravitational field divided by the rate at which it is created by mean shear.

Figure 1 compares the variation of the normal stresses with R_f according to (3.1) and Webster's (1964) experimental data.

From (3.1) a stable gravitational field leads to a relative† gain in the level of streamwise fluctuations at the expense of the vertical ones. The relative magnitude of the lateral fluctuations is unaffected. These features are generally in accord with Webster's measurements. It ought to be said that Webster feared that his data never attained equilibrium; certainly this may be the case for the separation of the normal stresses at $R_f = 0$ is substantially less than that obtained by Champagne *et al.* (1970). The level of agreement between predicted and measured behaviour is thus probably satisfactory.

Let us note that, at the other extreme, in a very unstable flow where R_f is large and negative, the ratios of the normal stresses are as follows:

$$\overline{u_1^2}:\overline{u_2^2}:\overline{u_3^2} = 0{\cdot}53:0{\cdot}53:0{\cdot}94.$$

That is, the relative stress levels of $\overline{u_1^2}$ and $\overline{u_3^2}$ are exactly reversed from their non-buoyant values.

Let us now examine the equations for the non-zero shear stress and heat fluxes given by (2.10) and (2.16). In (2.16) we now replace ϕ'_T by $\frac{1}{2}\phi_T$ since c_{2T} is taken as 0·5.

$$-\overline{u_1u_3} = \phi\frac{k\overline{u_3^2}}{\epsilon}\frac{\partial U_1}{\partial x_2} - \phi\alpha\frac{\overline{u_1T'}gk}{\epsilon T}, \tag{3.2}$$

$$-\overline{u_1T'} = \phi_T\frac{k}{\epsilon}\left(\overline{u_1u_3}\frac{\partial T}{\partial x_3} + \tfrac{1}{2}\overline{u_3T'}\frac{\partial U_1}{\partial x_3}\right), \tag{3.3}$$

$$-\overline{u_3T'} = \phi_T\frac{k\overline{u_3^2}}{\epsilon}\frac{\partial T}{\partial x_3} + 0{\cdot}8\phi_T\alpha\frac{k^2}{\epsilon^2}\frac{\overline{u_3T'}}{T}g\frac{\partial T}{\partial x_3}, \tag{3.4}$$

where g denotes the acceleration due to gravity. Equation (3.4) may be directly rearranged as follows:

$$-\overline{u_3T'} = \gamma\frac{\overline{u_3^2}k}{\epsilon}\frac{\partial T}{\partial x_3}, \tag{3.5}$$

where
$$\gamma \equiv \phi_T(1 + 0{\cdot}8\phi_T B)^{-1} \tag{3.6}$$

and B, a dimensionless buoyancy parameter, is defined by

$$B \equiv \frac{\alpha k^2 g}{\epsilon^2 T}\frac{\partial T}{\partial x_3}. \tag{3.7}$$

While, from a computational point of view, B is a more useful parameter than R_f (since B alone affects the value of γ), it contains quantities that are not usually, nor easily, measured in buoyancy-affected turbulence. Equation (3.6) is therefore modified to bring the flux Richardson number into prominence. First, the mean temperature gradient is eliminated by use of (3.5), and thus

$$\gamma = \phi_T\bigg/\left(1 - \frac{0{\cdot}8}{\gamma}\frac{k}{\overline{u_3^2}}\frac{\alpha\overline{u_3T'}g}{T\epsilon}\right). \tag{3.8}$$

Now it is readily shown that

$$-\alpha\overline{u_3T'}g/T\epsilon = \frac{R_f}{1-R_f}\frac{P}{\epsilon}.$$

† The absolute magnitude of k, and likewise of the individual stress components, will diminish, however.

Thus, after eliminating $k/\overline{u_3^2}$ by means of (3.1), noting that P and ϵ are equal for the case considered and rearranging (3.8) to give an explicit formula for γ, there results

$$\gamma = \phi_T(1{\cdot}59 - 5{\cdot}22R_f)/(1{\cdot}59 - 2{\cdot}82R_f). \tag{3.9}$$

From (3.9), the value of γ falls to zero when R_f reaches 0·305, which seems satisfactorily close to the value of 0·35 reported by Webster (1964).

To assist study of the equations for the shear stress and horizontal heat flux, a dimensionless coefficient β is introduced:

$$-\overline{u_1 u_3} \equiv \beta \frac{\overline{u_3^2}k}{\epsilon}\frac{\partial U_1}{\partial x_3}. \tag{3.10}$$

Equation (3.3) may then be recast as

$$-\overline{u_1 T'} = \phi_T(1 + 0{\cdot}5\gamma/\beta)\frac{\overline{u_1 u_3}k}{\epsilon}\frac{\partial T}{\partial x_3}, \tag{3.11}$$

and, in turn, (3.11) is used to eliminate $\overline{u_1 T'}$ from (3.2). After some manipulation the result may be expressed as

$$\beta = \phi/[1 + \phi\phi_T(1 + 0{\cdot}5\gamma/\beta)B]. \tag{3.12}$$

Then, on dividing (3.12) by (3.6) and noting that the quantity β/γ is more usually called the turbulent Prandtl number σ_t, we may obtain

$$\sigma_t = \sigma_{t0}\{[1 + \phi_T(0{\cdot}8 - 0{\cdot}5\phi_T)B]/(1 + \phi\phi_T B)\}. \tag{3.13}$$

σ_{t0} stands for ϕ/ϕ_T, which equals the turbulent Prandtl number under non-stratified conditions; for the present values of the empirical coefficients $\sigma_{t0} = 0{\cdot}63$, which is indeed typical of the values of the turbulent Prandtl number found in non-buoyant free shear flows.

From (3.13) it is seen that as B tends to infinity, that is under strongly stable conditions, σ_t tends to $0{\cdot}8/\phi_T - 0{\cdot}5$, so the turbulent Prandtl number is about 2·0. Thus vertical momentum transport is inhibited rather less severely by the gravitational field than is the vertical heat flux. The measurement of σ_t is always rather imprecise because of the large amount of data processing needed. Webster's data at two different sections gave values differing by a factor of two though the variation with Richardson number was nearly the same for the two cases. Figure 2 therefore shows the variation of σ_t/σ_{t0} as a function of the local 'gradient' Richardson number Ri ($\equiv \sigma_t R_f$); agreement between experiment and prediction is as close as may be expected in the circumstances.

The temperature–velocity correlation coefficients are easier to measure. From (2.15) and (3.5) it is readily deduced that

$$-\frac{\overline{u_3 T'}}{(\overline{u_3^2})^{\frac{1}{2}}\,(\overline{T'^2})^{\frac{1}{2}}} = (\gamma/1{\cdot}6)^{\frac{1}{2}}, \tag{3.14}$$

while, by introducing (3.5) and (3.10) into (3.11), we obtain

$$\frac{\overline{u_1 T'}}{\overline{u_3 T'}} = -\frac{\phi_T}{\gamma}[1 + \tfrac{1}{2}\sigma_t^{-1}]\left(\beta\frac{k}{\overline{u_3^2}}\bigg/(1 - R_f)\right)^{\frac{1}{2}}. \tag{3.15}$$

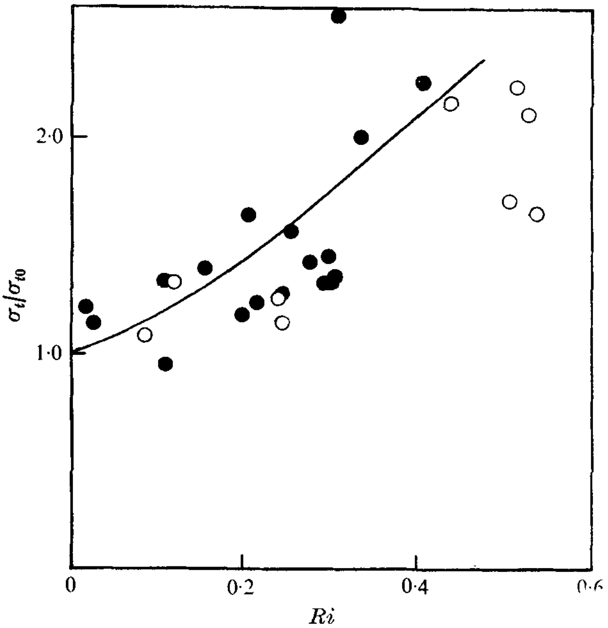

FIGURE 2. Turbulent Prandtl number in a stably stratified flow.
○, ●, Webster's data at two stations.

The predicted correlation coefficients for the vertical and horizontal heat fluxes are shown in figures 3 (*a*) and (*b*), comparison again being made with Webster's data. For stable flow the calculated coefficient for the vertical heat flux falls steeply as *Ri* increases while that for $\overline{u_1 T'}$ rises gradually. The behaviour is again in line with experiment. Although the present model is strictly inapplicable to flows near walls (because of the neglect of the influence of a wall on the pressure scrambling terms in (2.1) and (2.2)) it is known that these correlation coefficients are relatively unaffected by the presence of a wall. Some experimental results by Zubkovsky & Tsvang (1966, reported by Yaglom 1969) obtained in unstable conditions in the lower regions of the atmospheric boundary layer have therefore been included in figures 3 (*a*) and (*b*). The predicted variation shows the same trends as the measurements, though the experimental data suggest a slightly greater dependence of the correlation coefficients on *Ri* than is implied by the model. This small discrepancy could well be due to the presence of the ground.

The final two figures bring out the different effects that a stable buoyancy field has on the momentum and heat transport processes. It is seen from figure 4 that the shear-stress correlation coefficient is virtually constant for *Ri* up to 0·25. In contrast the ratio $-\overline{u_3 T'}/\overline{u_1 T'}$, shown in figure 5, falls from 0·9 under neutral conditions to 0·4 over the same span of Richardson numbers. As in the earlier comparisons, Webster's experimental data, though understandably showing scatter, certainly support the behaviour predicted by the model.

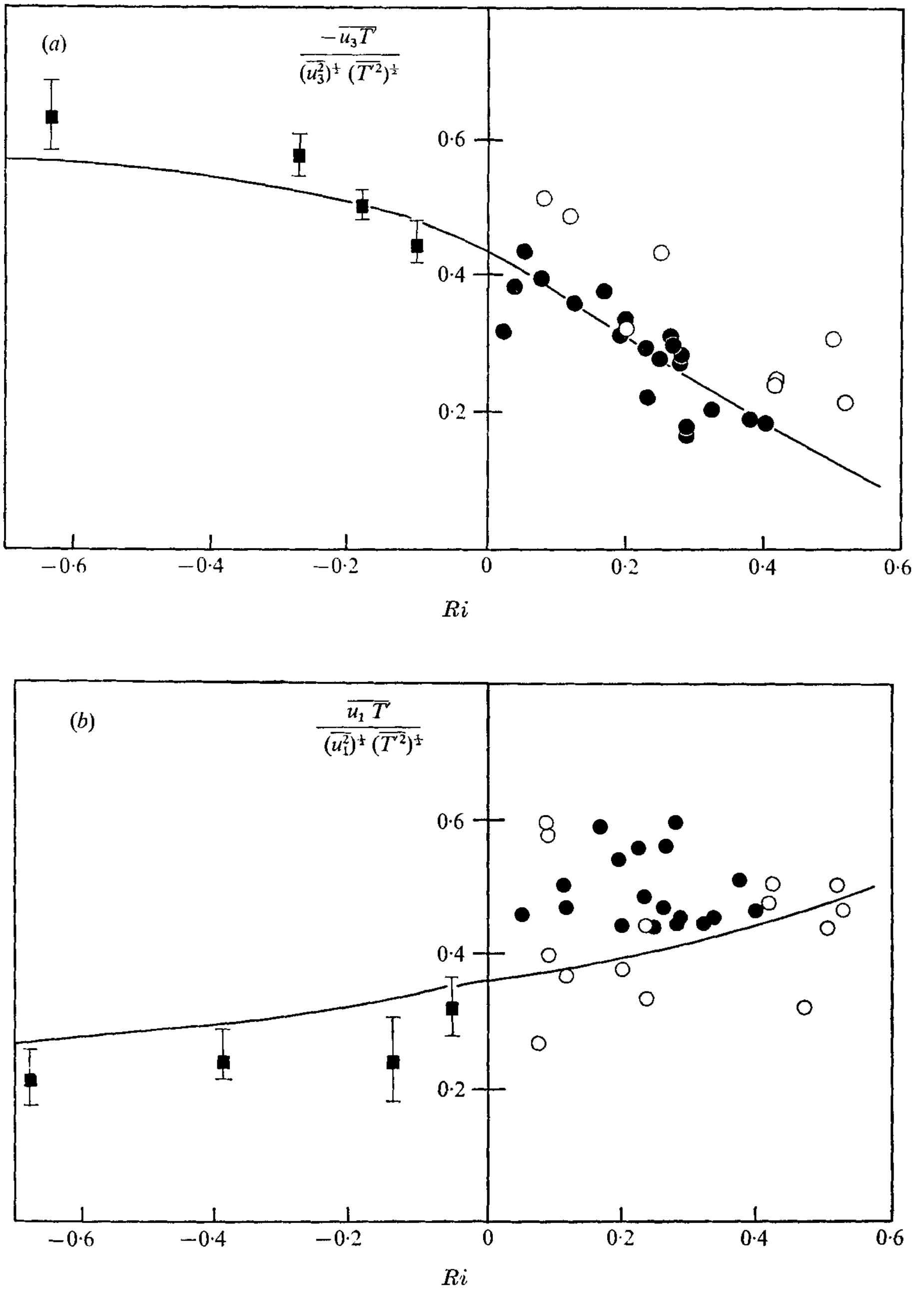

FIGURE 3. Variation of (*a*) vertical and (*b*) horizontal heat-flux correlation coefficient with Richardson number. ○, ●, Webster's data at two stations; ■, Zubkovsky & Tsvang (1966); bars indicate range of recorded values.

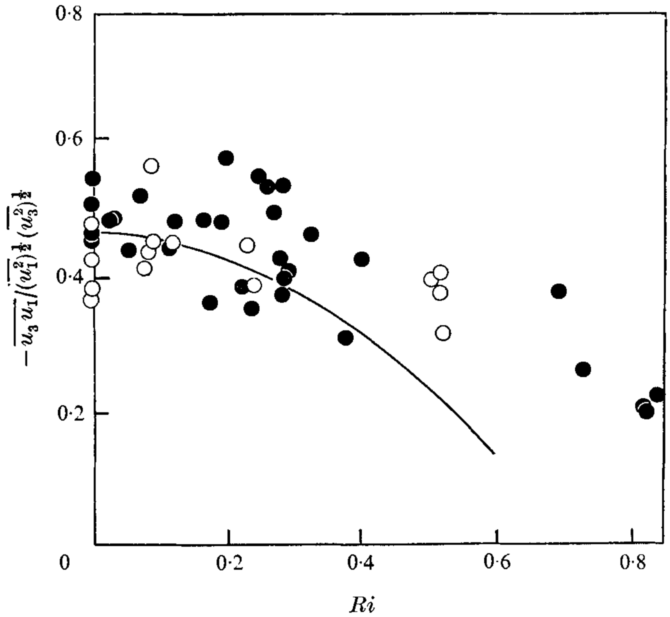

FIGURE 4. Shear-stress correlation coefficient in stably stratified flow. ○, ●, Webster's data at two stations.

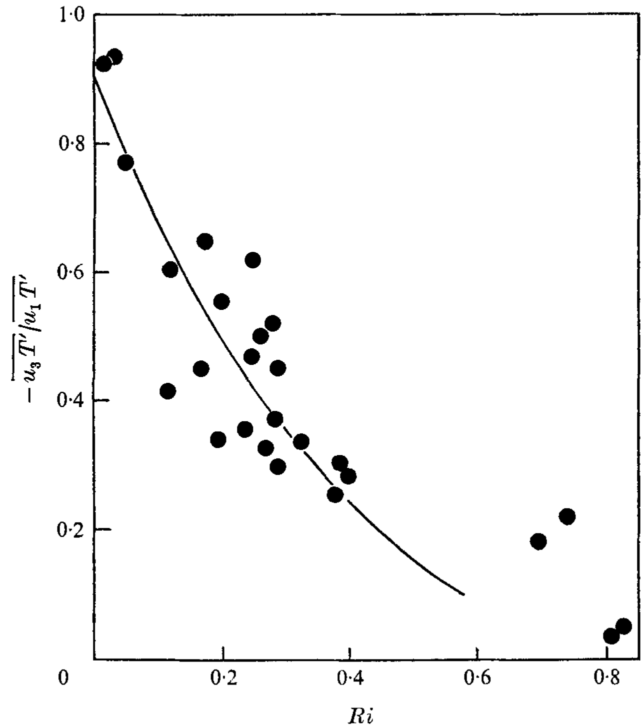

FIGURE 5. Ratio of vertical to horizontal heat flux in a stably stratified flow. ●, Webster's data.

4. Concluding remarks

The main proposition of the present note is that one of the actions of pressure fluctuations in a turbulent flow is to make the effective stress- and heat-flux-generation tensors more isotropic. Although no direct experimental confirmation is available, the linear hypothesis assumed in the analysis leads to predicted effects of buoyancy on the turbulence structure in general agreement with the trends measured by Webster (1964). The extension of the present proposal to the case of other kinds of force fields is obvious.

The most apparent limitation of the model is that it applies only to flows remote from walls. The presence of a horizontal wall diminishes vertical velocity fluctuations (and enlarges those parallel to the mean flow). Consequently, the critical Richardson number is lower; almost certainly less than 0·1 in near-wall flow (e.g. Nichol 1970) and only about 0·15 in the core region of a pipe (e.g. Ellison & Turner 1960) where the flow is still significantly under the influence of the wall (cf. Bradshaw 1973; Launder *et al.* 1975). For the same reason, the ratio $-\overline{u_1T'}/\overline{u_3T'}$ for wall turbulence is about 3·0 under neutral conditions compared with the value of 1·1 in the present work.

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